Mathematics, says an outstanding civil engineer, is a language, a tool, and a game. Also it can—and should—be fun.

Math's a Pleasure

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I HATED mathematics. It was a symbol of everything that oppressed me in my childhood. I was afraid of it and resented it. Not until I psychoanalyzed myself by taking a Ph.D. in it did I overcome the fear and, with it, the resentment.

When I was a boy we lived for several years in Spain, and my parents decided to teach me at home. Mother provided most of the instruction, but for mathematics Father took over. He was a very positive man—the embodiment of authority. Every day I sat before him while he lectured at me, filled a blackboard with strange marks in chalk, and asked me questions which required exact answers. He had a lot of fun. I suffered.

My suffering, I realized later, is still felt by most youngsters who face the terrors of mathematics—terrors compounded of infallibility, incomprehensibility, and authority in an amalgam that is thoroughly inhuman. The fact that it is one subject in which parents and teachers insist upon perfection adds to its grim repute.

"How much is seven and five?" a young mother of my acquaintance asked her small daughter, who was being initiated into the earliest mystery of mathematics—arithmetic.

"Eleven," the child suggested after some hesitation.

"No!" came the stern, uncompromising reply. "It's twelve."

"Well, I was close, wasn't I, Mamma?" the little girl pleaded hopefully.

Of course she was close. But that didn't appease her mother. A little wrong is all wrong when it comes to mathematics—at least according to the average parent and teacher. That little girl will have to penetrate fairly far into the higher reaches of the subject to find out that these absolutes are false; that five and seven may not always be exactly twelve; that three times five is fifteen, but five times three may turn out to be something called 15a.

Meanwhile she is well on the road to believing that mathematics is simply a matter of learning facts which have no meaning, and remembering them long enough to pass an examination. I did that myself. I even did it well enough to get A plus in calculus. I can still recall my despair when a friend of the family congratulated me warmly, adding: "We need good mathematicians."

The prospect of that A plus forcing me into mathematics as a career gave me the sinking sensation of stepping onto apparently solid ground and having it suddenly give way under my feet. Actually I had no idea what the calculus was all about.

IF MATHEMATICS were used only to discipline the mind—a meaningless concept I have sometimes heard teachers defend—this lack of understanding about it might not be so bad. But in fact mathematics is the common language and common tool of all science, upon which our world is coming to rely more and more. We live in a civilization that demands the mass production of scientific achievement. This in turn demands mathematics.

Even among college-trained men and women—perhaps I should say especially among them—I find a tendency to argue that, since understanding the science by which we live depends upon mathematics, they'll just take literature or art and leave science to the scientists. This defeatist attitude is induced by early training. For a child it is easier to understand the calculus than Shakespeare. Mathematics calls only for imagination and an ability to play games—the qualities in which children are pre-eminent—but Shakespeare demands an experience and understanding of life which no child can have. Pascal at thirteen had taught himself all the mathematical knowledge of his time, but he was a long way from reaching the peak of his talents as a literary philosopher.

A Matter of Rebellion

PERHAPS he would not have found mathematics so easy if he had not taught himself. For, although the subject calls primarily for the exercise of imagination, it is usually taught as though every mathematical theory were an absolute fact which no one dares question. In this country students are invited to form their own opinions and reach their own conclusions about English, history, and the humanities generally. In the physical sciences, even when there is little freedom in the teaching, the student can observe the results of experiments. In mathematics there is nothing to watch (which is why it calls for imagination) and the neophyte is discouraged from exploring the unseen for himself.

The result is a sort of schizophrenia in which mathematics takes to itself all the burdens of resented authority. American children especially resent authority, and rebel against it pretty much in proportion to the amount of it they get at home. Psychologists have noted—without always explaining it—that American children from authoritarian homes usually have more trouble with mathematics than their fellows. Boys especially identify mathematics with their fathers.

"I just don't get long division" is easier for a child to say than "I hate my father."

A similar psychological barrier is imposed on the children of "over-loving" mothers.

European schools used to be credited with teaching mathematics more skillfully than their American counterparts. Certainly the average graduate seemed to have a better grasp of the subject and less fear of it. But that was primarily because, in countries like Germany, everything was taught in the same manner, and the student was treated consistently by all his teachers as a person needing authoritative guidance.

In fourteen years of teaching engineering students, who are supposed to have some background in mathematics by the time they reach graduate school, I have found that 90 per cent are pitifully unprepared, and have to be taught all over again from an entirely new point of view.

These students come to us accomplished in mathematics exactly as deaf children who have been taught to play the piano might come to a conservatory of music. You could teach the deaf children just which keys to strike—and enforce the lesson by rapping them over the knuckles when they struck a false note. They would *understand* music as well as most of our students *understand* math. They probably would be just as fond of it, too.

DESPITE the great demand for engineers and the obvious attraction of the profession, only 8 per cent of the high-school graduates who qualify take up engineering. Why? In a survey which sought the answer, an overwhelming majority of the students gave as their first reason: "I'm afraid of math."

By the time they got into high school their imaginations had, in most instances, been killed. The youngsters who preserved imagination up to this point and kept it through high school against all odds turned to the arts rather than to engineering.

This attitude toward mathematics is so common it has become socially acceptable. The same bright person who boasts in company that he "could never understand math" would be ashamed to acknowledge that he doesn't understand Shakespeare or Freud. This attitude, too, has created the myth about mathematicians' extraordinary intelligence. To the layman any professor of mathematics is a genius—even if a slightly despicable one.

Having experienced these sentiments myself, I have tried to develop courses which restore imagination, remove the authoritarian complex, and relate math to life.

The first step in this direction is to rid the student of a lot of misconceptions. Anyone who can read can do the same thing for himself, but before he starts, he should understand what mathematics is and is not. It is a language, a tool, and a game—a method of describing things conveniently and efficiently, a shorthand adapted to playing the game of common sense or logic, as it is called in scientific circles. It is a human phenomenon, *not* an infallible proof of anything.

The Discovery of Numbers

MATHEMATICS came into being when some primitive genius discovered that counting could be done in the abstract. Before his day, men had been able to count, say, three stones, or three tigers, or three trees. Our genius, in a great burst of imagination, conceived of three as an abstract number and found that he could apply it to anything and to all things.

This is a step most of us are capable of today, even after the worst our schools can do to us. But some people never grasp even this elementary abstraction properly. They are like the members of a class of student nurses, whose sad story I heard recently. In order to be able to mix medicines accurately, these girls were taught for a whole term about fractions, with pints, quarts, and gallons of milk used as examples. In their final examination they were asked to calculate the mixture they would get if they mixed a quart of vanilla ice cream with two-thirds of a quart of chocolate. Seventy per cent of the class failed. Their imaginations did not stretch from milk to ice cream.

However, enough men learned to count satisfactorily to serve the needs of civilization for many thousands of years. For a long time men made things, even the pyramids, without fixed rules of measurement. It was not until builders had been using a right angle to square off their walls for several thousand years that a Greek named Euclid came along and put it all down in the form of the abstract geometry, with points so small and lines so thin that they take no space. (Incidentally, this is usually the form of mathematics that Americans grasp most easily—partly because it can be visualized and partly because the Greeks were superb teachers.)

A few thousand years later another genius, this time unknown, took another formidable step. Instead of thinking about twos and threes and twenties, he became interested in what you could do with *all* numbers in general—in, in other words, algebra. Algebra is a shorthand used to arrive at answers to general problems instead of solving each separately by the boring and inefficient process of plain arithmetic. We owe our knowledge of it to the Arabs, who either invented or transmitted it to us, along with the symbols for our numerals.

Primitive man and even the highly civilized man of antiquity had little use for precise measurements of speed, but some three hundred years ago the demand arose for a good description of how—and how fast—bodies move. In response to this demand, the Englishman Newton and the German Leibniz, quite independently of each other, invented the branch of mathematics we know as differential and integral calculus.

It is worth noting that almost nothing new in mathematics ever was deduced until there was a practical use for it. But as soon as there was a need, the answer came swiftly. Twenty years ago, when the progress in aviation demanded a method for solving equations having to do with the performance of airplanes, an American and an Englishman, unknown to each other, came up with the same answer. Their essentially identical works were published the same week.

Mathematics is the language that has made possible the rapid progress in modern science. In the past, scientific strides were made by a solitary genius here and there unveiling a bit of the mystery of the world. Today quite ordinary fellows, working in teams, communicate through mathematics and, using mathematics as a tool, bring about tremendous technological advances.

ONCE you have got rid of the idea that mathematics is a set of absolute and arbitrary relationships, the next step is to realize that it is a game—and a game for which man made the rules. The student can too if he likes.

When my child was three he invented his

own numerical system, which went: one, two, three, four, six, seven, eight, nine, and ten. Five did not exist. There was nothing "wrong" with my child's game, provided you played it his way and not ours. It is great fun to play games in a slightly different way from the accepted ones; all children do it, and that is why, if they are unspoiled by formal education, they can do so well at mathematics.

Games are played for mere fun; mathematics can also be played for intellectual satisfaction or for practical results. A good way to get rid of the fear and hate of mathematics is to apply it to problems in which you are deeply interested.

One of the thoroughly despised parts of college mathematics is called "inverse trigonometric functions." Students hope to be able to remember these well enough to pass the examination; they do not care to understand them. When this point is reached in one of my courses, I introduce it something like this:

"Gentlemen, I suppose we all would rather be going to the movies than sitting in this classroom. Let us imagine, therefore, that we are doing just that. Let us imagine, furthur, that we have our choice of seats anywhere in the house.

"How can we decide just where we will get the best view of the screen?"

This is a problem that makes sense. By the time the class has taken all the factors into account and discovered the best vantage point in a particular theater, the students have also grasped the meaning of inverse trigonometric functions.

This is how mathematics came into being. Men set themselves a problem imaginatively and solved it. The great mathematicians were much more akin to poets than to pedants. They were fallible, too. Most of the papers published by the leading authorities in the field contain some mistakes in mathematics. Mistakes are not fatal.

One of the best ways to learn mathematics is to discover its truths for yourself. A few teachers are beginning to realize this; they have known all about it in the humanities for a long time.

NOT long ago a child in the third grade asked her teacher: "Why can't I divide by zero?" It's a good point, but by the time she is fifteen, the child won't ask such questions. In this case her teacher unhappily did not know the answer, and did not like to give the usual one: "You can't, that's all," or, "Because the book says so."

This teacher felt guilty and ashamed, and when she met me she asked me. I countered with another question: "How much is one hundred divided by fifty?"

"Two, of course."

"And by five?"

"Twenty."

"And by one?"

"One hundred."

"And by one-half?"

"Two hundred."

"And by one-tenth?"

"One thousand."

"And by one-hundredth?"

"Ten thousand. . . . Oh, I get it. The smaller the number you divide by, the larger the result. So when you divide by zero you get . . . infinity!"

The young teacher found the answer by herself. In doing so she also discovered an application of the idea of limit and obtained a better grasp of the concept of infinity. And she did all this in two minutes, in a manner she could use with her own grade students.

The right approach to mathematics, then, is to unbridle the imagination. But this can only be done after the mistaken ideas and feelings about it have been cleared away.

One Man's Goose Is Another Man's Yam

oc·a·ri·na (ok'ə-rē'nə), ***n.*** [It., dim. of *oca*, a goose; L. *auca*, a goose: so called from its shape], a small, simple wind instrument shaped like a sweet potato. . . .

—*Webster's New World Dictionary of the American Language,* College Edition, 1953.